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# Theoretical and experimental modelling of an open oscillatory heat pipe including gravity

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#### **Abstract**

An open oscillatory heat pipe is a two-phase flow device capable of transferring heat from a source to a sink below the source, against the force of gravity, without the aid of a wick or any moving mechanical parts. A theoretical model of such a device taking gravity, surface tension, friction and pressure into account is presented. The model uses vapour bubble, liquid plug and liquid film control volumes. An experimental model was constructed and tested using water as the working fluid. It was found that the device could operate indefinitely provided the heat source is less than 30 mm above the heat sink and that the temperature of the heat source is less than 160 °C. The theoretical model was able to predict these experimentally determined values. By calculating Lyapunov exponents it is shown that the theoretical model is able to reflect the characteristic chaotic behaviour of these devices. It was concluded that the model represents the experimental situation well and that it is important to consider the evaporation of liquid deposited on the surface by the trailing edge of the liquid plug. It is recommended that convective heat transfer is further investigated and the water pumping ability of the device is exploited. 2003 Elsevier SAS. All rights reserved.

*Keywords:* Open oscillatory heat pipe; Pulsating heat pipe; Mathematical modelling; Chaos theory; Lyapunov exponent; Water pumping

### **1. Introduction**

An oscillatory or pulsating heat pipe (PHP) is a two-phase flow device used for transferring heat without any moving mechanical parts [1,2]. A particularly simple variant of such a device, termed an open oscillatory heat pipe (OOHP), may consist of a length of relatively long but small diameter pipe bent in the middle to form a loop as shown in Fig. 1. If the pipe is initially filled with water, the one end heated and the other ends cooled, oscillatory pulses are observed in which water is expelled from the open ends and then drawn back into the pipe in a seemingly periodic fashion. The diameter of the pipe is important. It must be small enough such that under operating conditions a liquid plug and vapour bubble type flow pattern occurs. If the diameter is too large the tendency is for a stratified flow pattern to occur with the liquid flowing in the bottom of the pipe and the vapour in the top.



Fig. 1. Example of an OOHP.

This type of open oscillatory heat pipe is deemed important for a number of reasons. It is a simple example of the more complex case of oscillatory or pulsating heat pipes (PHPs) in which the pipe meanders back and forth many times between the heated and cooled ends and in which there are many bubbles and plugs and for which a satisfactory theoretical model has as yet not been published [3]. The open

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*Greek symbols*

## **Nomenclature**



*δ* thickness . m *θ* liquid-surface contact angle . . . . . . . . . . . . . . . <sup>o</sup> *<sup>ρ</sup>* density . kg·m−<sup>3</sup> *<sup>µ</sup>* viscosity . kg·m−1·s−<sup>1</sup> *φ* angle to the horizontal . ◦ *<sup>τ</sup>* shear stress. N·m−<sup>2</sup>  $\sigma$  surface tension . . . . . . . . . . . . . . . . . N·m<sup>-1</sup> *Subscripts* a adiabatic c condenser, cold e exit, environment, evaporator f film, friction g gravity i inlet f friction, film L leading  $\ell$  liquid  $\ell f$  liquid film P pressure p plug T trailing v vapour *τ* shear stress

oscillatory heat pipe can thus be used to study the physical behaviour of this more complex system. The open oscillatory heat pipe is able to transfer heat relative to gravity from a heat source positioned above the cold sink without the use of a wick or any moving mechanical parts. The open oscillatory heat pipe is also able to propel a boat as well as able to pump water. Although as a thermodynamic heat engine its efficiency for doing work is extremely small, conversely, as a heat transfer device it is thus necessarily very efficient.

The object of this paper is to investigate both experimentally and theoretically the influence of gravity on the motion of this one vapour bubble and two liquid plug system OOHP by varying the vertical distance between the heated end and the water level in the cold water tank.

Simple mathematical models for bubble-plug flow in a OOHP and/or a PHP have been proposed [4–7]. Somewhat more advanced or complicated models that include the effect of a liquid film [8–10] have also been proposed. The chaotic behaviour of oscillatory heat pipes has been considered in the literature [11–13]. Attempts at correlating experimentally determined heat transfer characteristics with dimensionless parameters such as Bond, Froude, Weber, Prandtl and Kutateladze numbers have also been undertaken [14,15]. There is also a tendency to undertake more experimental studies in an attempt to better understand the operating regimes of PHPs [16,17].

Because of the seemingly random and aperiodic behaviour (of for instance the position of the liquid plug  $x_p$  relative to its initial position in the open oscillatory heat pipe) a times series analysis will be undertaken and the Lyapunov exponents calculated. Given some initial condition  $x_0$ , consider a nearby point  $x_0 + \delta$ , where the initial separation  $\delta_0$ is extremely small. Let  $\delta_n$  be the separation after *n* steps. If  $|\delta_n| \approx |\delta_0|e^{n\lambda}$ ,  $\lambda$  is called the Lyapunov exponent. A positive Lyapunov exponent is a signature of chaos. A more computationally useful formula may be derived [18] as

$$
\lambda = \lim_{n \to \infty} \left[ \frac{1}{n} \sum_{i=0}^{n-1} \ln \left| f'(x_i) \right| \right] \tag{1}
$$

For stable fixed points and cycles, *λ* is negative; for chaotic attractors, *λ* is positive.

## **2. Theoretical modelling**

*σ* surface tension w water, wall

Three types of one-dimensional control volumes as shown in Fig. 2 are used to theoretically model the physical behaviour of the oscillatory heat pipe. A control volume represent-



Fig. 2. Theoretical model of the OOHP.

ing the vapour bubble entrapped in the closed end, a control volume representing a liquid plug moving back and fourth at the open end, and an annular control volume representing the thin liquid film deposited on the inside surface of the pipe by the trailing end of the liquid plug.

The rate of change of mass of the vapour bubble with time depends on the net mass flow rate

$$
\frac{\Delta m_{\rm v}}{\Delta t} = \dot{m}_{\rm vi} - \dot{m}_{\rm ve} - \dot{m}_{\rm v, dep} \tag{2}
$$

The mass flow rate of vapour entering the vapour bubble is by evaporation of the liquid film in the evaporator

$$
\dot{m}_{\rm vi} = \dot{m}_{\ell \rm fe} = U_{\rm i} \pi \, d \, L_{\rm ev} (T_{\rm e} - T_{\rm v}) / i_{\rm fg} \tag{3}
$$

*L*ev is the length of the liquid film in the evaporator that is in contact with the vapour bubble and  $U_i$  is a characteristic overall heat transfer coefficient for heat transfer from the heat source into the liquid film. Similarly, the rate at which mass is leaving the vapour bubble by condensation on the cooled length in contact with the vapour is given by

$$
\dot{m}_{\text{ve}} = \dot{m}_{\ell\text{fi}} = U_{\text{e}}\pi dL_{\text{vc}}(T_{\text{v}} - T_{\text{c}})/i_{\text{fg}}
$$
(4)

Under certain conditions the temperature of the vapour may rise above the heat source temperature due to the work done by the liquid plug on the vapour as it moves towards the closed end of the pipe. Under this condition

$$
\dot{m}_{\rm v,dep} = U_{\rm ve} \pi d L_{\rm v,dep} (T_{\rm v} - T_{\rm e}) / i_{\rm fg}
$$
\n<sup>(5)</sup>

The rate of change of mass of the liquid film similarly depends on the net mass flow

$$
\frac{\Delta m_{\ell f}}{\Delta t} = \dot{m}_{\ell f i} - \dot{m}_{\ell f e} + \dot{m}_{\ell f, dep} \tag{6}
$$

At the trailing edge the liquid plug deposits a layer of liquid of thickness  $\delta_{\ell f}$  on the pipe wall as it moves towards the open end of the pipe. This rate of liquid deposited is given by

$$
\dot{m}_{\ell f, \text{dep}} = \rho_{\ell} \pi d \delta_{\ell f} \nu_p \tag{7}
$$

The mass of a liquid plug is given in terms of the bubble length  $x_p$  and the pipe half-length  $L/2 = L_e + L_a + L_c$  as

$$
m_{\rm p} = \rho_{\ell} \pi d^2 (L/2 - x_{\rm p} + L_{\rm m})/4
$$
 (8)

The forces acting, in the direction of motion, on the liquid plug are due to shear (friction), gravity, surface tension, and vapour bubble and external water pressures and the equation of motion is

$$
m_{\rm p} \frac{\Delta v_{\rm p}}{\Delta t} = -F_{\tau} \pm F_{\rm g} - F_{\sigma} + F_{\rm P}
$$
\n(9)

The forces are

$$
F_{\rm g} = \rho_{\ell} L_{\rm p}^{\phi} \pi d^2 g \sin \phi / 4 \tag{10}
$$

 $L_p^{\phi}$  is the portion of the liquid plug inclined at angle  $\phi$  to the horizontal

$$
F_{\sigma} = \pi d\sigma (\cos \theta_{\text{T,L}} - \cos \theta_{\text{L,T}})
$$
\n(11)

$$
F_{\rm P} = \pi d^2 (P_{\rm v} - P_{\rm e}) / 4 \tag{12}
$$

$$
F_{\tau} = \tau \pi dL_p = C_f \rho_{\ell} \pi dL_p v_p^2 / 2 \tag{13}
$$

The coefficient of friction is (conveniently) approximated by  $C_f = 0.078Re^{-0.25}$  for  $Re > 1180$  or  $16/Re$  for  $Re < 1180$ , where  $Re = \rho_{\ell} v_{\rm p} d / \mu_{\ell}$ .

The bubble and liquid film momentum are both relatively small compared to the liquid plug momentum and are neglected.

The internal energy and hence the temperature of the vapour bubble as a function of time depends on the net convective heat transfer, the net enthalpy, and the rate at which the bubble is doing work on the liquid plug and is given (ignoring the kinetic and potential energies) by

$$
\frac{\Delta E_{\rm v}}{\Delta t} = \dot{q}_{\rm vi} - \dot{q}_{\rm ve} + \dot{m}_{\rm vi}i_{\rm vi} - \dot{m}_{\rm ve}i_{\rm ve} + P_{\rm v}A \frac{\Delta x_{\rm p}}{\Delta t} \tag{14}
$$

where  $\Delta E_v = m_v c_{vv} \Delta T_v$ ,  $i_v \approx 2500 + c_{pv} T_v$ ,  $A = \pi d^2/4$ ,  $\dot{q}_{\text{vi}} = h_{\text{e}} \pi dL_{\text{ev}} (T_{\text{we}} - T_{\text{v}})$  and  $\dot{q}_{\text{ve}} = h_{\text{c}} \pi dL_{\text{vc}} (T_{\text{v}} - T_{\text{wc}})$ .

The vapour bubble is assumed to be an ideal gas and hence if the volume and temperature are known the pressure must be given by

$$
P_{\rm v} = \frac{m_{\rm v} R_{\rm v} (T_{\rm v} + 273.15)}{\pi d^2 x_{\rm p} / 4}
$$
\n(15)

In Eq. (15) the effect of the relatively thin liquid film has been ignored when determining the volume of the vapour bubble. The temperature of the vapour in Eq. (15) has been converted to Kelvin by adding 273.15 to the Celsius value.

The net thrust or the force restraining the pipe from moving in the axial direction is calculated by

$$
R = 2\left[ (P_v - P_e)\pi d^2/4 \mp F_\tau \right] \tag{16}
$$

There are two open ends and hence the value "2" in Eq. (16).  $F<sub>\tau</sub>$  is given by Eq. (16) and its sign depends on whether the liquid plug is moving out or into the pipe, and  $F_{\sigma}$  may be neglected because it is relatively small (see Fig. 8).

An explicit finite difference numerical scheme is used to solve this set of equations. The annular liquid film control volume of length *L*<sup>e</sup> is divided into a number *N* of smaller control volumes of length *L*e*/N* to account for evaporation, condensation as well as the deposition of liquid *δ*<sup>f</sup>*,*dep as the liquid plug moves out of the evaporator. The effect of a thin liquid film in the adiabatic and condensing sections *L*<sup>a</sup> and *L*<sup>c</sup> was neglected. The time steps used in the solution of the numerical equations were reduced until stable and repeatable results were obtained. The number of smaller liquid film control volumes in the evaporator was varied and the results tested for consistency. A time step of not more than 0.0001 s and liquid film control volume lengths of about 0.005 were found to give a good balance between computer running times and numerical accuracy.

#### **3. Experimental set-up and results**

A special open oscillatory heat pipe was manufactured and orientated (in a stand) as shown in Fig. 3. It shows half of the vapour bubble the one liquid plug (of the two) in the symmetrical half of the pipe of total length  $2 \times$  $930 = 1860$  mm and internal diameter 3.34 mm. The central 360 mm of the pipe is bent (like a snake) and cast into a  $20 \times 110 \times 150$  mm aluminium block to ensure good thermal contact between the pipe and the block. The temperature of the aluminium block is heated and controlled by means of an electrical resistance wire and a variable voltage supply. The two open ends are inserted in a large tank of cooling water. The aluminium-heating block is mounted on a calibrated thrust sensor. The pipe is filled with water, the aluminium block heated and the magnitude of the resulting oscillatory thrust imparted to the block is measured as well as the heating block temperature and the water temperature. Five thin *K*-type thermocouples each inserted into narrow evenly spaced holes in the heating block are used to determine its temperature.

A typical experimentally determined curve of thrust as a function of time is given in Fig. 4 and may be compared with the typical theoretically determined thrust curve shown in

Fig. 5. The basic numerical values of the variables and initial values used in the computer program are given in Table 1. In both Figs. 4 and 5 the thrust oscillates at a frequency of 5.5 Hz and intermittent transient decaying pulses occur at somewhere between 3 and 10 second intervals. These pulses occur when the liquid in the heated evaporator section dries



Fig. 3. Symmetrical half of an OOHP.



Fig. 4. Example of an experimentally determined thrust curve for the open oscillatory heat pipe, (b) is the thrust for the first 5 s of (a) but on a larger time-scale, and (c) is the thrust for the time period of 20 to 25 s of (a).



Fig. 5. Example of a theoretically determined thrust curve for the open oscillatory heat pipe, (b) is the thrust for the first 5 s of (a) but on a larger time-scale, and (c) is the thrust for the time period of 20 to 25 s of (a).





out (normally the vapour bubble and liquid plug interface would be oscillating in the cooling section of the pipe at a frequency of 5.5 Hz). When evaporator dry-out occurs no additional mass is added to the portion of the vapour bubble exposed to the heated surface, vapour however continues to condense in the portion exposed to the cooling surface; there is a sharp decrease in the vapour bubble pressure and the outside environment pressure forces the liquid plug into the evaporator section. In this way the dry evaporator is recharged with liquid. The liquid plug position, vapour bubble pressure and mass of liquid in the film in the evaporator during this process is shown in Fig. 6.

The experimental and theoretical thrust results may be analysed using chaos theory. This is done to establish whether the response curves are cyclical or non-cyclical and hence chaotic. Lyapunov exponents as defined by Eq. (1) are calculated for both the theoretically and experimentally determined thrust response curves. Fig. 7 shows the calculated Lyapunov exponents as a function of the temperature of the heating block *T*e. It is seen that experimentally measured thrust curves (of which Fig. 4 is an example) have positive Lyapunov exponents of about 3 at the lower temperatures but increases to about 4 at  $170^{\circ}$ C; thus illustrating the increasingly chaotic nature of the oscillations as the temperature increases. Lyapunov exponents for theoretically determined thrust curves (of which Fig. 5 is one example) were positive albeit somewhat lower than for the experimental components. Although the theoretical values are seen to be lower than the experimental values they do exhibit the same tendency of increasing as temperature increases. The positive theoretical values and this increasing tendency with temperature does however demonstrate that the theoretical model is able to capture the chaotic behaviour of an OOHP. Three of the points determined from experimental response curves are noticeably lower than rest of the points. The reason for this is that these three points are based on experimental response curves during which there were relative long times when no thrust was detected. Eq. (1) interprets zero thrust as being non-chaotic and hence the lower values.

Fig. 8 gives an idea of the relative magnitude of the forces, in Eq. (9), that act on the liquid plug. The pressure



Fig. 6. Liquid-plug position, vapour-bubble pressure and evaporator film mass as a function of time during an evaporator recharging process.



Fig. 7. Lyapunov exponents as a function of temperature.



Fig. 8. Pressure force difference, and friction, gravity and surface tension forces ( $F_{\sigma}$  is in mN).

force dominates, the friction and the gravity forces are significant but the surface tension force (in mN) is about three orders of magnitude smaller. Although the force in the axial direction due to surface tension is negligible the surface tension is necessary for the formation of the liquid plugs and vapour bubbles in the pipe.

During operation the heating block is positioned above the level of the water in the tank. Not only can the OOHP transfer heat from a heat source position above the heat sink, it can also draw water up against gravity into the evaporator. The vertical distance between the heating block and water



Fig. 9. Theoretically calculated liquid plug positions  $x_p$  and thrust *R* as a function of time (in seconds) for different values of: vertical distance between evaporator and water level  $L_w$ , film thickness deposited at the trailing end of a liquid plug  $\delta_{\ell f}$  and evaporator temperature  $T_e$  of the base-case values given in Table 1.

level was varied. It was experimentally determined that the OOHP could operate for an indefinite period of time only if this height was less than 30 mm. The theoretical model was able to simulate this behaviour as shown by Fig. 9(a).

Provided the heating block temperature did not exceed about  $150\,^{\circ}\text{C}$  the OOHP would always work indefinitely and would always start working again if left to cool down for a couple of days. However as the temperature increased to above  $170\degree C$  the OOHP would stop working; it could be restarted at this temperature by manually recharging the evaporator with liquid but would only work for a few minutes before stopping again. The effect of increasing the evaporator temperature is illustrated in Fig. 9(c). Fig. 9(b) gives an idea of the effect of the thickness of the liquid film deposited by the trailing end of the liquid plug; as expected, the thicker the film the less often the evaporator needs recharging.

## **4. Discussion**

Convective heat transfer to and from both the vapour bubble and the liquid plug was not included in the theoretical model. Convective heat transfer terms were included in the vapour bubble energy equation  $(Eq. (14))$  but the heat transfer coefficients were taken as zero in the numerical solutions. A single control volume has different portions of it surface exposed to both heating and cooling temperatures along the pipe length. To take this temperature variation into account would require using more than one control volume and possibly unduly complicate the model but needs further investigation.

Heat transfer to the liquid plug needs to be addressed. Boiling of water in glass tubes tends to demonstrate that when vapour is formed in the liquid a single vapour bubble it quickly formed within the liquid plug, expands rapidly (due to evaporation of the thin liquid film) and forming two liquid plugs on either side of itself. Also, in glass pulsating heat pipes the system of liquid and vapour plugs tend to oscillate slightly before moving more rapidly back and forth between the heated and cooled ends. It is thus suggested that the dominant heat transfer mode between the heated and cooled ends of a PHP is due to the evaporation and condensation of vapour from and to a liquid film and not due to heating and cooling of liquid plugs. The heating of a liquid plug is important up until a vapour bubble is formed and which then subsequently grows rapidly due to the evaporation of the liquid film between the growing bubble and the heated surface; resulting in the characteristic rapid movement of all the other bubbles and plugs. The other cause for the characteristic rapid movement in a pulsating heat pipe could be due to the dry-out of the liquid film and the resulting rapid decrease in pressure as characteristic of the OOHP as shown in Fig. 6.

In a separate experimental set-up the heating block was positioned below the water tank and the two pipes passing thought sealed holes in the bottom of the tank and the open ends below the water level. Under these operating conditions the heat pipe could be operated indefinitely, even when the cooling water in the tank approached its boiling point. (In this bottom heat mode and with the tank water at boiling point the characteristic oscillatory behaviour of two liquid plugs in unison was no longer apparent but rather liquid continued to be drawn into the one tube with vapour flowing out of the other open end.) This tended to show that although the OOHP could only pump water a vertical height of about 30 mm in top heat mode it potential as a water pump in bottom heat mode is clearly demonstrated.

### **5. Conclusions**

- The mathematical model reflects the physical behaviour of an open oscillatory heat pipe.
- The surface tension plays a vital role in ensuring the formation of liquid plugs, the dominant forces affecting liquid plug movement are however the vapour and ambient pressure, the friction and the gravitational forces.
- To theoretically model pulsating heat pipes the evaporation and condensation of the liquid-film between the bubbles and the wall must be included in the formulation of the model.
- The open oscillatory heat pipe is not suitable for pumping water more than 30 mm in the top heat mode, however in the bottom heat mode it can pump water much higher that 30 mm and needs to be further investigated.
- Heat transfer between the heated and cooled ends of a pulsating heat pipe is due to film evaporation and condensation; characteristic sudden movement of the system of plugs and bubbles is due to rapid bubble growth and rapid bubble collapse.

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